\[ R \approx 8 \, J/(K \cdot mol) \approx 0.08 \, L \cdot atm/(K \cdot mol) \]

1 bar = 10^5 Pa \quad 1 \text{ bar} = 1.0 \text{ atm}

\[ \Delta U = q_{by} + w_{on} = q_{by} - \int P_{ext} dV \]

\[ H = U + PV \]

\[ C_v = \left( \frac{\partial U}{\partial T} \right)_V \]

\[ C_p = \left( \frac{\partial H}{\partial T} \right)_p \]

\[ \mu_{JT} = \left( \frac{\partial T}{\partial P} \right)_H \]

\[ \kappa_T = -\frac{1}{V} \left( \frac{\partial V}{\partial P} \right)_T \]

\[ P_1 V_1^\gamma = P_2 V_2^\gamma, \quad T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \quad \text{adiabatic process} \quad \gamma = \frac{C_p}{C_v} \]

\[ \frac{1}{(1-x)^4} \approx 1 + 4x + 10x^2 + 20x^3 \cdots \quad \forall x < 1 \]

\[ Z = \frac{p}{\rho RT} = 1 + B_2 \rho + B_3 \rho^2 + \cdots \quad \rho = \frac{n}{V} \]
1. (10 pts) The density of a gas, presumed to be ideal, is 1.0 g/L at 300K and 190 torr. What is the molar mass of the compound?

\[ PV = nRT, \quad n = \frac{g}{M_w} \]

\[ PV = \frac{g}{M_w} RT \quad \text{or} \quad M_w = \left( \frac{g}{V} \right) \frac{RT}{P} = \frac{1g}{L} \cdot \frac{0.08L\cdot atm}{300K \cdot \text{mol} \cdot K} \]

\[ M_w = \frac{4 \times 0.08 \times 300}{760} = 8 \times 3 \times 4 \times \frac{g}{\text{mol}} \]

\[ = 96 \text{ g/mol} \]

2. (10 pts) If the Joule-Thomson coefficient \( \mu_{JT} = 1.25 \text{ mK/bar} \), what pressure differential is needed to cause a 10 K drop in temperature?

\[ \mu_{JT} = \left( \frac{\Delta T}{\Delta P} \right)_{H} \quad \text{or} \quad \Delta P = \frac{\Delta T}{\mu_{JT}} = \frac{10K}{1.25 \times 10^{-3} \text{ K/bar}} \]

\[ = 8000 \text{ bar} \]
3. Consider a fluid whose pressure obeys

\[ P = \frac{\rho RT}{(1 - \frac{1}{4}b \rho)^4} - a \rho^2 \]

where \( a, b \) are constants. Calculate

(a) (10 pts) the second and third virial coefficients;

\[
\frac{P}{\rho RT} = \frac{1}{(1 - \frac{1}{4}b \rho)^4} - \frac{a}{RT} \rho
\]

\[ \approx 1 + 4x + 10x^2 + \ldots \quad \text{with} \quad x = \frac{1}{4}b \rho \]

\[ Z = 1 + b \rho - \frac{a}{RT} \rho + 10\left(\frac{1}{4}b \rho\right)^2 + \ldots \]

\[ \therefore \quad B_2 = b - \frac{a}{RT}, \quad B_3 = \frac{10}{16}b^2 = \frac{5}{8}b^2 \]

(b) (5 pts) and, the Boyle temperature (at which the second virial coefficient vanishes).

\[ B_2 = 0 \Rightarrow b = \frac{a}{RT_B}, \quad T_B = \frac{a}{RB} \]
4. One mole of nitrogen gas \((C_v = (5/2)R)\) with an initial temperature of 300 K expands reversibly from an initial volume of 1 L to 10 L.

<table>
<thead>
<tr>
<th></th>
<th>(\Delta H \text{ (kJ)})</th>
<th>(\Delta U \text{ (kJ)})</th>
<th>(q_{by} \text{ (kJ)})</th>
<th>(w_{by} \text{ (kJ)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>0</td>
<td>0</td>
<td>2.4 \ln(10)</td>
<td>2.4 \ln(10)</td>
</tr>
<tr>
<td>(b)</td>
<td>-5.0</td>
<td>-3.6</td>
<td>0</td>
<td>3.6</td>
</tr>
</tbody>
</table>

(a) (12 pts) Calculate the entries in the Table if the process is isothermal.

\[
\Delta U = q_{by} - \int p dV \Rightarrow q_{by} = \int p dV = nRT \int \frac{dV}{V} = nRT \ln \left( \frac{V_2}{V_1} \right)
\]

\[
q_{by} = RT \ln(10) = 8 \frac{J}{\text{mol} \cdot \text{K}} \cdot 300 \text{ K} \cdot \ln(10) = 2.4 \times 10^3 \text{ J} \ln(10)
\]

\[
\Delta H = -W_{by} = 0
\]

(b) (12 pts) Calculate the entries in the Table if the process is adiabatic. Hint: \((1/10)^{2/5} \approx 0.4\)

\[
\Delta H = C_p \Delta T, \quad \Delta U = C_v \Delta T = -W_{by}, \quad q_{by} = 0
\]

but \(\left( \frac{T_2}{T_1} \right)^{\frac{C_p}{C_v}} = \left( \frac{V_1}{V_2} \right)^{\frac{1}{C_v}} = \left( \frac{1}{10} \right)^{2/5} = 0.4\), so \(T_2 = T_1 \cdot 0.4 = 120 \text{ K}\)

\[
\Delta T = (120 - 300) \text{ K} = -180 \text{ K}
\]

\[
\Delta U = \frac{5}{2} \cdot 8 \frac{J}{\text{mol} \cdot \text{K}} \cdot (-180 \text{ K}) = -3.6 \text{ kJ}
\]

\[
\Delta H = \frac{5}{2} \cdot 8 \frac{J}{\text{mol} \cdot \text{K}} \cdot (-180 \text{ K}) = -5.0 \text{ kJ}
\]

(c) (6 pts) In which of the two expansions does the gas do the most work and why?

Isothermal since heat is constantly supplied to the gas.
5. (10 pts) Prove that $C_p - C_v = R$ for one mole of an ideal gas, stating all assumptions and starting from the definition of the enthalpy.

$$\Delta H = dU + d(PV) = (C_v + nR) \, dT = C_p \, dT$$

$$\int_{C_v} \, dT \quad \text{and} \quad nRT$$

$$\therefore \quad C_p = C_v + R \quad \text{for} \quad n = 1$$

6. (10 pts) When 200 J of energy are supplied to 2.0 moles of Ar under conditions of constant pressure, its temperature rises by 5 K. What are the molar heat capacities, $C_v$ and $C_p$ in units of $R$?

$$Q_p = C_p \Delta T, \quad C_p = \frac{Q_p}{\Delta T} = \frac{200J}{5K \cdot 2\text{mols}} = 20J \quad \text{K} \cdot \text{mol}^{-1}$$

$$C_v = C_p - R = (20 - 8) J/K \cdot \text{mol} = 12 J/K \cdot \text{mol}$$
7. (10 pts) Show that

\[ dq_{rev} = dH - VdP \]

starting from the First Law of Thermodynamics for a reversible process.

\[
dH = dU + PdV + VdP, \quad dU = dq_{rev} - PdV \quad \text{for a reversible process} \\
\]

\[
dH = dq_{rev} - PdV + PdV + VdP \\
\therefore dq_{rev} = dH - VdP 
\]

8. (10 pts) $dq_{rev}/T$ is an exact differential. Express $dq_{rev}/T$ as a function of $H$ and $P$, and from this result, determine

\[
\left( \frac{\partial(1/T)}{\partial P} \right)_H = \ldots \\
\]

\[
\frac{dq_{rev}}{T} = dH \left( \frac{1}{T} \right) - \left( \frac{V}{T} \right) dP \\
\text{since } dq_{rev}/T \text{ is exact, then} \\
\left( \frac{\partial(1/T)}{\partial P} \right)_H = - \left( \frac{\partial H \left( \frac{V}{T} \right)}{\partial H} \right)_P 
\]